

Comment on “Scaling of the linear response in simple aging systems without disorder”

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We have repeated the simulations of Henkel, Paessens, and Pleimling (HPP) [Phys. Rev. E **69**, 056109 (2004)] for the field-cooled susceptibility $\chi_{FC}(t) - \chi_0 \sim t^{-A}$ in the quench of ferromagnetic systems to and below T_C . We show that, contrary to the statement made by HPP, the exponent A coincides with the exponent a of the linear response function $R(t, s) \sim s^{-(1+a)} f_R(t/s)$. We point out what are the assumptions in the argument of HPP that lead them to the conclusion $A < a$.

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In a recent paper [1] Henkel, Paessens, and Pleimling (HPP) have addressed the question of the relationship between the scalings of the linear response function and the zero-field-cooled (ZFC) susceptibility in ferromagnetic spin systems undergoing aging after a quench to or below T_C . This problem (limited to the quenches below T_C) had been previously analyzed by us in a series of papers [2–5] with the following conclusions.

(1) The linear response function

$$R(t, s) = \left. \frac{\delta \langle \phi(t) \rangle}{\delta h(s)} \right|_{h=0}, \quad t \geq s \quad (1)$$

scales as

$$R(t, s) \sim s^{-(1+a)} f_R(t/s) \quad (2)$$

with

$$a = \frac{n}{z} \left(\frac{d - d_L}{d_U - d_L} \right), \quad (3)$$

where z is the dynamical exponent entering the growth law $L(t) \sim t^{1/z}$ of the average defect distance, d_L is the lower critical dimensionality, and $(n=1, d_U=3)$ and $(n=2, d_U=4)$ for scalar or vector order parameter, respectively.

(2) The ZFC susceptibility $\chi_{ZFC}(t, s) = \int_s^t ds' R(t, s')$ scales as

$$\chi_{ZFC}(t, s) \sim s^{-A} f_\chi(t/s) \quad (4)$$

with

$$A = \begin{cases} a & \text{for } d < d_U \\ n/z & \text{with log-corrections for } d = d_U \\ n/z & \text{for } d > d_U. \end{cases} \quad (5)$$

The cases considered by HPP in [1] correspond to $d < d_U$, where the above result gives $A = a$ [6]. Instead, HPP reach the different conclusion $A < a$, thereafter stating that A is a new exponent unrelated to a and to aging behavior. The purpose

of the present Comment is to show that the data for the same quench considered in the HPP paper, if properly interpreted, are in agreement with our finding $A = a$, where a is given by Eq. (3).

Following HPP, next to the ZFC susceptibility we introduce the field cooled (FC) susceptibility $\chi_{FC}(t) = \int_0^t ds R(t, s)$ and the thermoremanent (TRM) susceptibility $\rho_{TRM}(t, s) = \int_0^s ds' R(t, s')$. Obviously, the three integrated response functions satisfy the sum rule

$$\chi_{FC}(t) = \chi_{ZFC}(t, s) + \rho_{TRM}(t, s). \quad (6)$$

The HPP argument is built on the behavior of the FC susceptibility. From numerical computations they find

$$\chi_{FC}(t) = \chi_0 + \kappa t^{-A} \quad (7)$$

where κ is some constant. Making the distinction between systems of class S, with a finite equilibrium correlation length ξ , and systems of class L, with $\xi = \infty$, for χ_0 they make the statement

$$\chi_0 = \begin{cases} (1 - m_{eq}^2)/T & \text{for systems of class S} \\ 0 & \text{for systems of class L,} \end{cases} \quad (8)$$

where m_{eq} is the equilibrium magnetization at the temperature T . Then, assuming that the TRM susceptibility obeys the asymptotic behavior

$$\rho_{TRM}(t, s) = s^{-a} f_M(t/s) \quad (9)$$

from Eq. (6) follows

$$\chi_{ZFC}(t, s) = \chi_0 + \kappa t^{-A} - s^{-a} f_M(t/s), \quad (10)$$

where for the exponent a they take

$$a = \begin{cases} 1/z & \text{for class S} \\ (d - 2 + \eta)/z & \text{for class L.} \end{cases} \quad (11)$$

Let us now look separately to the different cases.

1. *Class S: 2d Ising model quenched below T_C .*

As an example of class S, HPP consider the $d=2$ Ising model with Glauber dynamics quenched below T_C . Measuring the FC susceptibility they find that Eq. (7) holds with χ_0 given by the top line of Eq. (8) and with $A=1/4$. Then, assuming that a is given by Eq. (11), that is by $a=1/z=1/2$, they make the statement $A < a$. According to them, A

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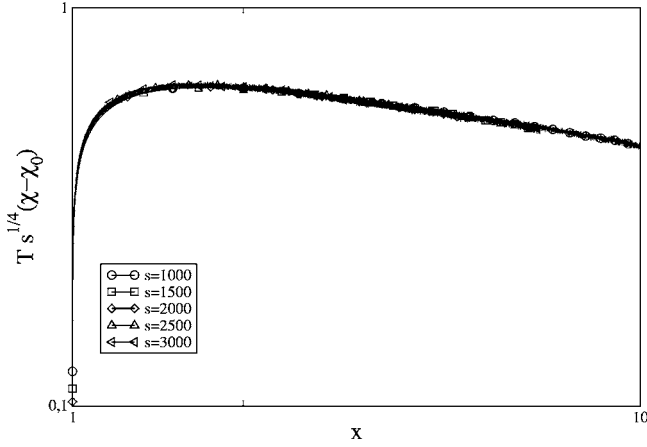


FIG. 1. Data collapse of $s^{1/4}[\chi_{ZFC}(t,s) - \chi_0]$ vs x .

does not have any relationship to aging and it is due to the roughness of the interfaces, while a is a subleading exponent.

Repeating the simulation of HPP with the same quench temperature $T=1.5$, we have reproduced their data for the FC susceptibility. However, rather than making an assumption on the value of a , we have looked for an unbiased comparison of A and a , born out of the same set of data. This can be accomplished by analyzing the ZFC susceptibility according to Eq. (10). In Fig. 1 we have plotted $s^{1/4}[\chi_{ZFC}(t,s) - \chi_0]$ as a function of $x=t/s$ for different values of s ranging from 1000 to 3000. The excellent data collapse in Fig. 1 has two possible origins: either $A < a$ and in the range of s considered the third term in the right-hand side of Eq. (10) is negligible, or $a=A$. In order to discriminate between these two possibilities it is enough to replot the same set of data as a function of t for fixed s . If the first alternative is the right one the data should collapse also in this plot, while in the second case there could be no collapse, due to the existence of the s dependence. Figure 2 shows that indeed the latter one is the case, that the s dependence is a large effect and that, therefore, $a=A=1/4$.

Although the pair of Figs. 1 and 2 is certainly enough to settle the issue, in order to help visualize the result we have also produced (Fig. 3) the log-log plot of $\chi_{ZFC}(t,s) - \chi_0$ as a function of s , for fixed x . The data in Fig. 3 display an ex-

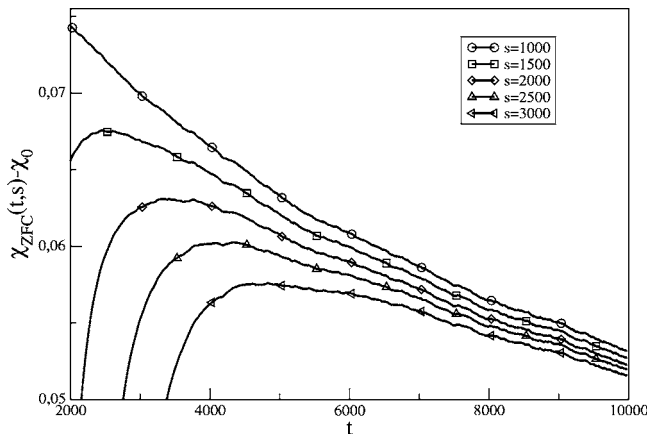


FIG. 2. $\chi_{ZFC}(t,s) - \chi_0$ plotted against t .

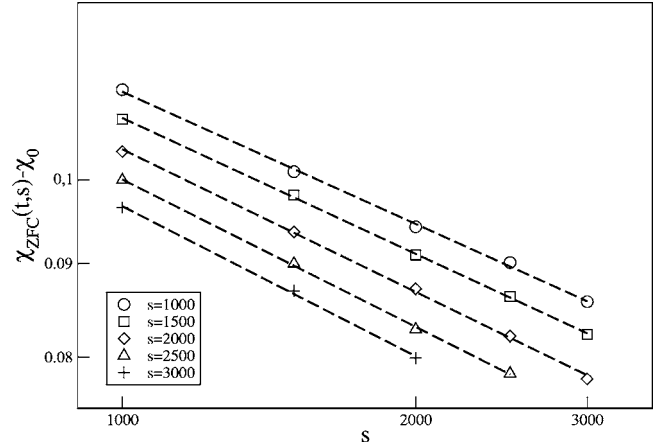


FIG. 3. $\chi_{ZFC}(t,s) - \chi_0$ plotted against s for different fixed values of x . Straight lines are the best fits (with exponents 0.24, 0.25, 0.25, 0.27, 0.27 from top to bottom).

cellent fit with a single power law, with a slope very close to $-1/4$ for every value of x , which makes clear at a glance the conclusion $a=A=1/4$ reached above and in agreement with Eqs. (3) and (5).

In summary, this means (i) that a is not subleading as believed by HPP and (ii) that roughening of the interfaces, rather than being unrelated to aging, is precisely the mechanism that renders the exponent a smaller (for $d < d_U$) than $1/z$, as explained in Ref. [5]. What goes wrong in the HPP interpretation of the data in [1] is the assumption that, for systems of class S, a is given by the top line of Eq. (11) [7].

2. Class L: 3d spherical model quenched to and below T_C .

As an example of class L, HPP consider the spherical model quenched to and below T_C . They compute numerically $\chi_{FC}(t)$ in both cases, with $d=3$, finding that it saturates to a constant for large t . However, they do not identify this constant with χ_0 , since they make the statement (8) that $\chi_0=0$ for systems of class L. This is supported by a hand waving argument according to which χ_0 ought to vanish, since the correlated clusters for systems of class L should have no “inside.” Although vague, this assumption is crucial because it is the starting point of the chain of implications used by HPP: the saturation of $\chi_{FC}(t)$ to a constant value and $\chi_0=0$ imply $A=0$, which in turn implies $A < a$, since in the spherical model $a=(d-2)/2$, both in the quench to and below T_C .

Before going further, it is necessary to clarify the physical meaning of the constant χ_0 appearing in Eq. (7). This can be readily understood recalling that

$$\lim_{t \rightarrow \infty} \chi_{FC}(t) = \chi_{eq} = (1 - m_{eq}^2)/T, \quad (12)$$

where χ_{eq} is the static susceptibility. Since the second equality in the above equation is nothing but the static fluctuation-dissipation theorem, it holds for all systems, be they of class S or class L, quenched below T_C or to T_C . Therefore from Eqs. (7) and (12), we can identify $\chi_0 = \chi_{eq}$ and Eq. (8) must be replaced by

$$\chi_0 = (1 - m_{eq}^2)/T \quad (13)$$

both for systems of class S and class L, which clearly reduces to $\chi_0 = 1/T_C$ for the quenches to T_C . Therefore $\chi_0 = 0$ assumed by HPP is excluded in all cases.

Furthermore, in the case of the spherical model there is no room for assumptions, since the model is exactly soluble. Using the formulas that HPP give in [1], it is not difficult to derive analytically the large t behavior of the FC susceptibility obtaining

$$\chi_{FC}(t) = (1 - m_{eq}^2)/T + \kappa t^{-(d-2)/2} \quad (14)$$

with $m_{eq}^2 = 1 - T/T_C$, $\kappa = [\Gamma(1 - d/2)\Gamma(1 + \omega/2)]/\Gamma(2 - (d - \omega/2))$ where Γ is the gamma function, $\omega = d/2 - 2$ in the quench to T_C and $\omega = -d/2$ in the quench below T_C . Therefore, comparing with Eq. (7), we have $\chi_0 = \chi_{eq} = (1 - m_{eq}^2)/T$ [8], as expected from Eq. (13). This implies $A = a = (d - 2)/2$ for $T \leq T_C$, in agreement with Eqs. (3) and (5).

3. Class L: 2d Ising model quenched to T_C .

As an additional instance of a system of class L, HPP consider the 2d Ising model quenched to T_C . Again, they find that the FC susceptibility saturates to a constant. By the same reasoning as in the previous case, from the assumption that χ_0 ought to vanish they make to descend $A = 0 < a = (d - 2 + \eta)/z_c = 0.115$, where we have used $z_c = 2.167$ [9] and the exact result $\eta = 1/4$.

Although the argument of Eq. (12) ought to suffice, we have repeated their simulations and we have plotted (Fig. 4) $\log_{10}(\chi_0 - \chi_{FC}(t))$ against $\log_{10} t$, with $\chi_0 = 1/T_C$. The figure shows a very clean power law decay with $A = 0.115 \pm 0.005$, which compares very well with the value of a given above

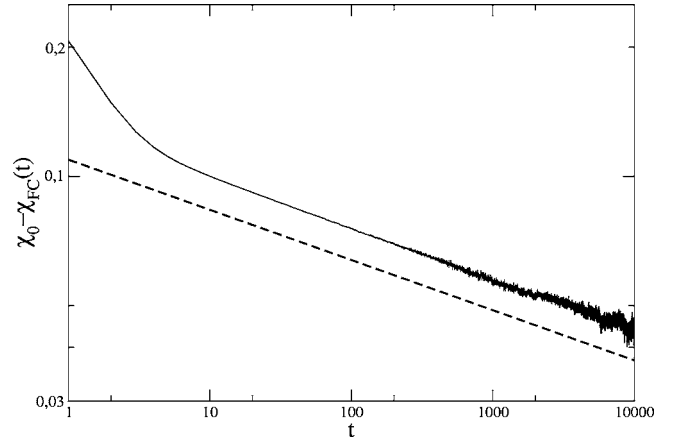


FIG. 4. The field cooled susceptibility $\chi_{FC}(t) - \chi_0$ vs t for the 2D Ising model at $T = T_C \approx 2.269$. The straight line is the expected behavior $\chi_{FC}(t) - \chi_0 \sim t^{\eta/z_c}$ with $\eta = 1/4$ and $z_c = 2.167$.

and yields, again, $A = a$. Furthermore, the observation of the decay with the correct value of the exponent implies that also the subtraction by $\chi_0 = 1/T_C$ is the correct one.

In summary, the unbiased analysis of the data for the FC susceptibility in all cases considered by HPP, that is in systems of class S and class L with $d < d_U$, yields $A = a$ in agreement with Eqs. (3) and (5). The biases in the HPP analysis, which lead to the wrong conclusion $A < a$, are in the two assumptions (i) $a = 1/z$ for systems of class S and (ii) $\chi_0 = 0$ for systems of class L.

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- [6] The general case, with the explanation of the mechanism producing the change of A from a to n/z above d_U , is analyzed in Ref. [4].
- [7] Here, one might wonder what is the justification for using the asymptotic form (9) in Eq. (10), when the TRM susceptibility is known [4] to be affected by a very long crossover. Using the HPP notation, let us rewrite $\rho_{TRM}(t, s)$ as the sum of the two contributions

$$\rho_{TRM}(t, s) = s^{-a} f_M(t/s) + s^{-\lambda R/z} g_M(t/s),$$

where $s^{-\lambda R/z} g_M(t/s) = r_1 t^{-\lambda R/z} = \int_0^t \epsilon ds' R(t, s')$, and t_ϵ is the characteristic time for the onset of scaling. The second contribution

on the right-hand side, with $\lambda_R/z > a$, is the correction to scaling responsible for the crossover. However, from Eq. (6) $\chi_{ZFC}(t, s) = \chi_{FC}(t) - \rho_{TRM}(t, s)$, it is evident that this term is canceled by the identical contribution in $\chi_{FC}(t)$ and, therefore, Eq. (10) holds also in the range of s reached in the simulations. Obviously, such a cancellation does not operate if the TRM susceptibility is observed alone. In that case, the second term in the displayed equation in this reference must be properly taken into account. This brings us to the question of the value of a , since HPP in [1] claim to have obtained $a = 1/2$ by fitting the data for the TRM susceptibility in the $d = 2$ Ising model. However, in order to obtain a good agreement with numerical data, they are forced to take a negative value $r_1 = -1.84$ for the parameter r_1 , while, by definition, $r_1 = t^{\lambda R/z} \int_0^t \epsilon ds' R(t, s')$ is a positive quantity. Therefore the estimation of $a = 1/2$ from TRM data is based on a nonphysical fit procedure.

- [8] Notice that, in the particular case of the spherical model, this gives $\chi_0 = 1/T_C$ for $T \leq T_C$, since thermal fluctuations are critical for all $T \leq T_C$.
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